co ti ity
o co p t l l ctio s

l o i
Mo cow i r ity

RL mi r
p rtm t of omput r ci c
or ll i r ity
M y ,

l :

. umbered st

. omput ble re l

3. omput ble m ppi

.

. s ts

N b r *is a ai X; e e X is a se and is a a ial a in f o na u al nu e s* **N** *on o X .*

N *P*

is a **n mb in** of *X* ,

i is a nu ber (or **ind x**) of *x* $x =$ (*i*

mpls :

$\mathbf{N}^2$ wi *i* − *!* *f* *i* ; *i*

i! *i* − *ign*(*i*) (*i* = *n i*

i! ' *i*

i W_i ' *i*

. C p t l ls

A co u able real is de ned below as e li i of a co u able se uence of ra ionals w ic is funda en- al in so e s andard way

A eal nu e a is a **p abl r al** *if e e is a e u si e fun ion* $'_i$ *su a fo all* $n; m\ 2\ \mathbf{N}$

$$j\mathbf{r}\ '_i\ n\ -\mathbf{r}\ '\ m\ {}^{-n}$$

and

$$a = \lim \mathbf{r}\ '_i\ n$$

Le y be e nu bering y :

A o u a le sequen e $c_0; c\ ; c_2; \ldots$ *in* *is*
p ably **a al** *if*

$$8n; m \; c_n - c_{n \; m} < \; ^{-n}$$

T o m: *E r com u b fun m n qu nc in con rg . or o r, h imi com u r c n b foun c i*

P oo . Li i algori : e li i co u able real is s eci ed by e se uence of ra ionals $r_0; r_1; r_2; \ldots$ suc a $_n$ is n 2nd ra ional a roxi a ion o $c_{n \; 2}$.

Le l be a recursive func ion w ic given an index i of a co u able funda en al se uence of reals i s li i $l(i)$.

. ist l s ts

A su se L X *fo a nu e ed se* X*;* [redacted] W_i

$$\bar{} \quad L = W_i \setminus$$ [redacted]

Index se of L is o en ially" r.e [redacted]

oms propertis of li t bls [redacted]

*is' **li abl** if fo so e .e. se* [redacted]

; X are lis able

ni e in ersec ions and r.e. unions [redacted] e 2s are lis able

do ain of every co u able a [redacted]

(Cf. X is r.e. X g [redacted]

inverse i ages of a lis able e [redacted] u able a ings

Li t bl st of comput bl r l :

$\{x \in \quad | \; x \quad 0\}$,

o en in ervals

$\{x \in \quad | \; x \neq 0\}$, e c.

o li t bl: closed in ervals, single ons, ra ionals, algebraic reals, irra ional 's, e c.

. M i

Se of ra ionals dense in every o en in erval

con ains a ra ional.

none lis a le se of o -

on ains a a ional

L

$i2$ **N**

y^{-} L $_i$ \

j n-th

and (0)

( )(2)

beaco u able funda en al se uence of ra ionals

converging o x be e s andard recursively [redacted]

enu erable nonrecursive se of na ural nu bers

$= f$

For eac n de ne a funda en al se uence $_n$

$$_n(k) = \begin{cases} q(m); & \textit{if n- h rogr m i o n} \\ & \textit{rmin in x c m k} \\ q(k); & \textit{o h rwi .} \end{cases}$$

if $n \not\in K$ en

$$_n = q(0); q( ); q(2); \ldots \longrightarrow x$$

if $n \in K$ en

$$_n = q \quad ; q( ); \ldots ; q(m); \ldots ; q(m); \ldots \longrightarrow q(m)$$

erefore $_n$ converges for eac n. Le $h(n)$ be e e ec ively co u ed index of e li i oin of $_n$. I is clear a

$$n \not\in K \Rightarrow h(n) \in W_i.$$

o e a (" does no old since is would give a nega ive es for K. erefore for so e $n \in K$ $h(n) \in W_i$ w ic gives a ra ional $q(m)$ in L

o olla t p oo L

6. Co ti ity o l ctio s

r E e eal o u a fue ion is

inuous

. Con(nui yof x) a omΨ:

$$\lim_{x \to a} \Psi(x) = \Psi(a)$$

on -

8k9m8 (& $\Psi(x)$) $j\Psi(x) - \Psi($ $)j <$ 2)

Sue ooseos

9m $(jx - aj <$ 2 & $\Psi(x)$ & $j\Psi(x) - \Psi(a)j$ 2^{-k})

Pick suc k and consider a se

$$Y = fyj \quad \Psi(y) \& \quad j\Psi(y) - \quad a)j > 2^{-k-1})g:$$

islis a esfinse lese

$$fiju - \quad \Psi(a)j > \quad 2^{k-1})g.$$

o a ion: $S(a;r)$ = in erval wi cen er and radius r

y e assu ions ade, e se s $U_m = Y \cap S(a; \; ^{-m-1})$ for all m are none y and lis able. y e ain le a, in eac U_m

q_m. Moreover, since $|q_m - a| < 2^{-m-1}$ e se- uence $\{q_0; q_1; q_2; \ldots\}$ is a co u able funda en al one converging o a. Consider

$$) \quad \Psi(a)| < 2^{-k \; 1})\}$$

V is a

. ff cti co ti ity

T o m (Se ara ion eore) *i n in ic of i -*
oin i b L_1 *n* L_2 *of com u b r n*
$_1$ *on c n c i n uch h*

(