— JProver —

An Efficient Refiner for First-order Intuitionistic Logic

Stephan Schmitt

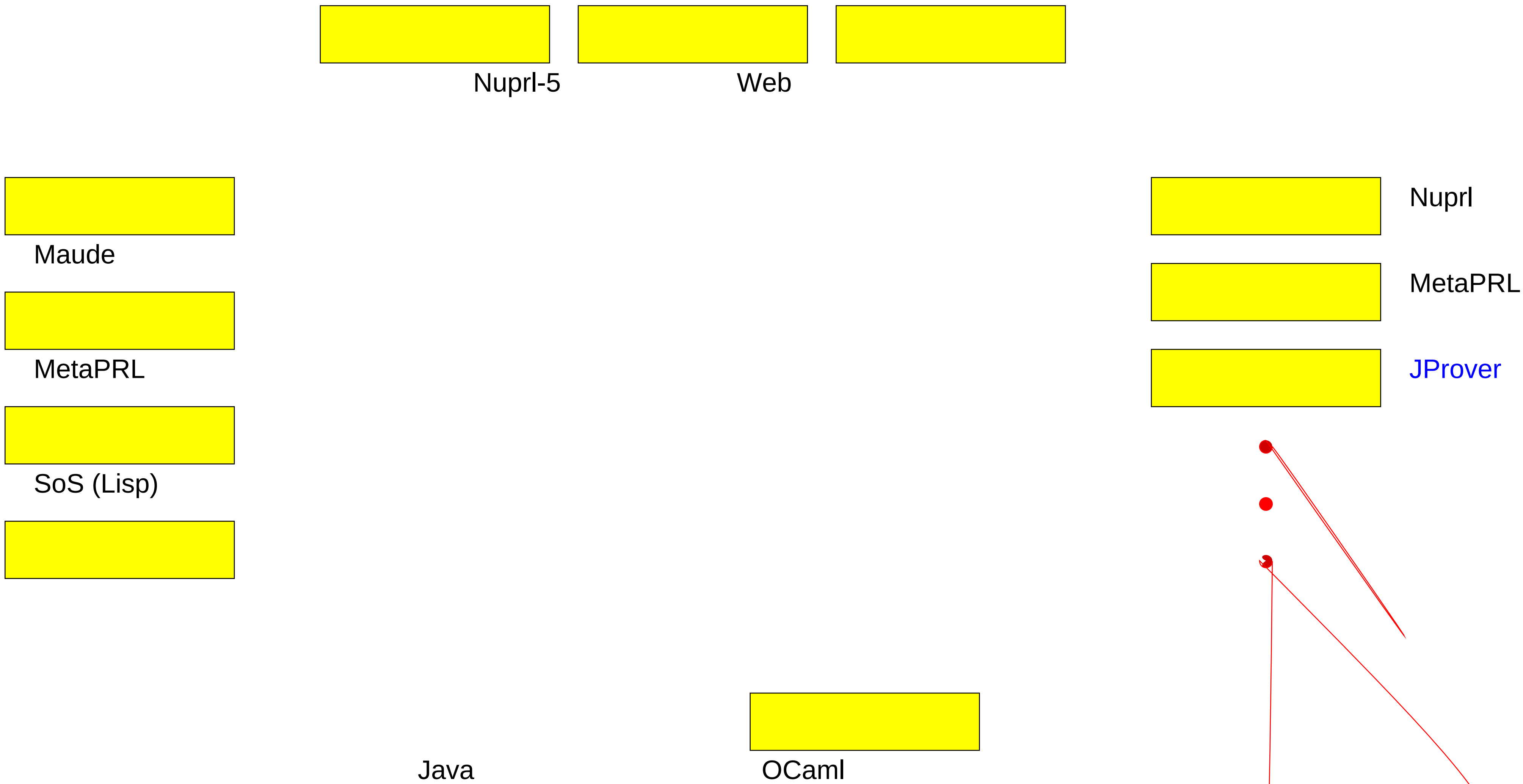
Department of Computer Science, Cornell University, Ithaca, NY, U.S.A.

steph@cs.cornell.edu

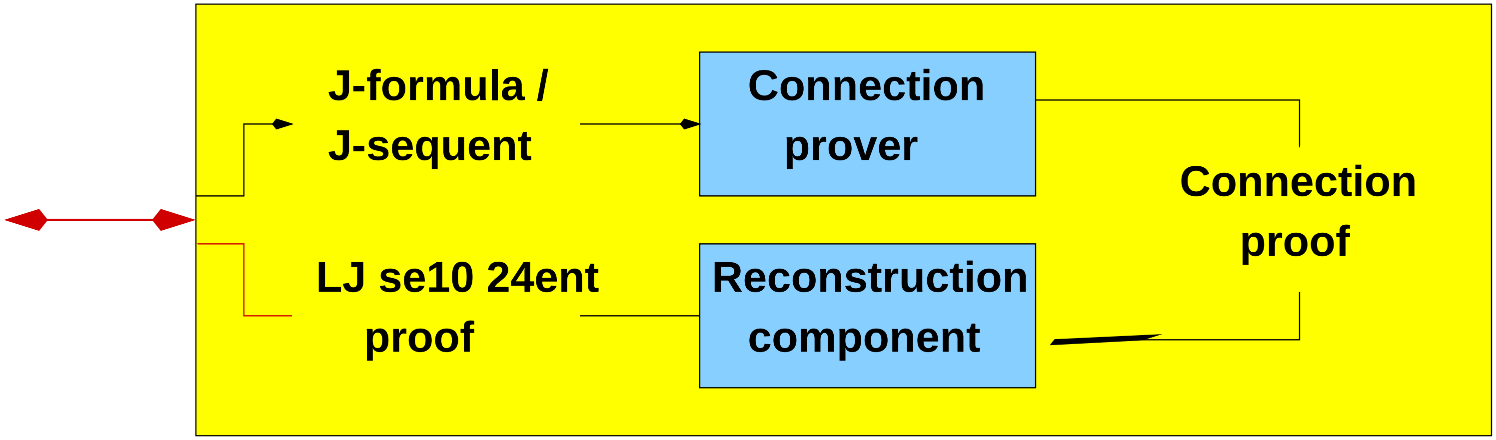
Motivation: Automated Theorem Proving

Connection to NuPRL-5

Concept: Add JProver as a new refiner to the NuPRL-5 architecture:



A CHITECTU E OF THE JProver REFINE



J-formula / J-sequen : Me aPRL erm, in erface be ween NuPRL and OCaml

Connec ion prover: Proof search s ra egy based on he ex ension procedure

(Bi el 1982, Otten & Kreitz 1995 – 2000)

uc ion componen : Search-free proof recons ruc ion procedure in o LJ or LJ_{mc}

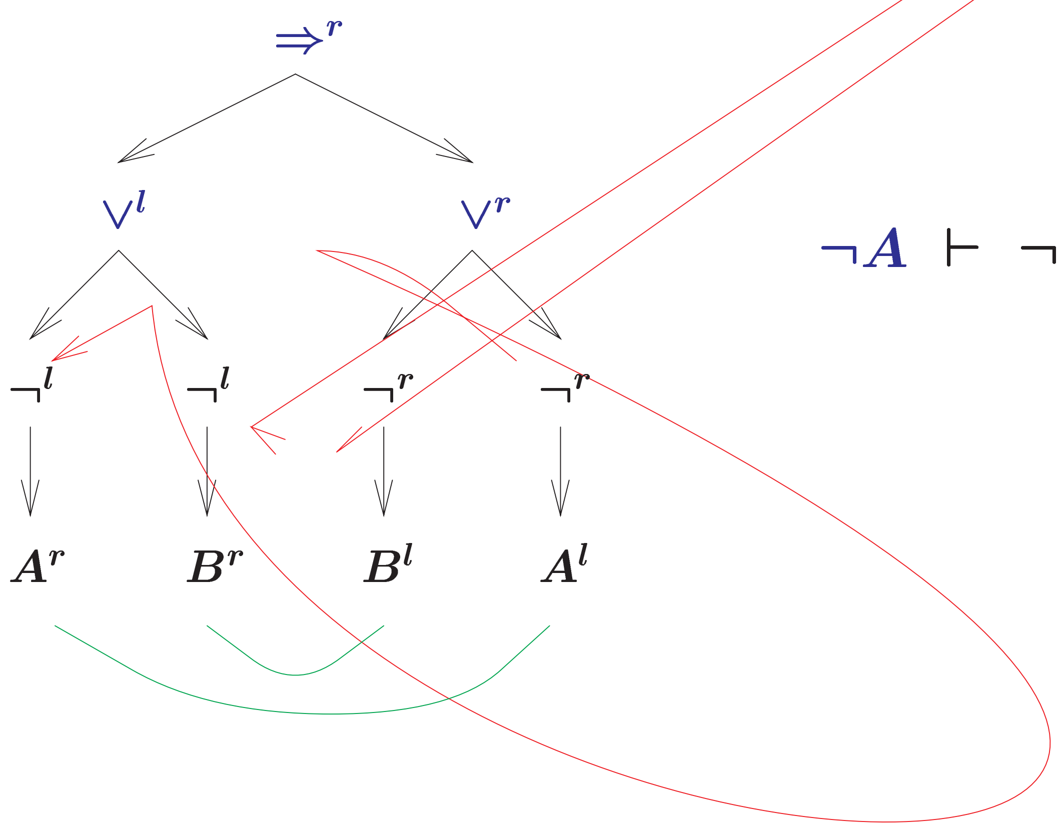
(Schmitt & Kreitz 1995 – 2000)

proof: Seen proof in Gen zen's single conclusioned seuen calculus LJ, i.e., he firs -order fragmen of ITT

P OOF RECONST UCTION I: EXAMPLE

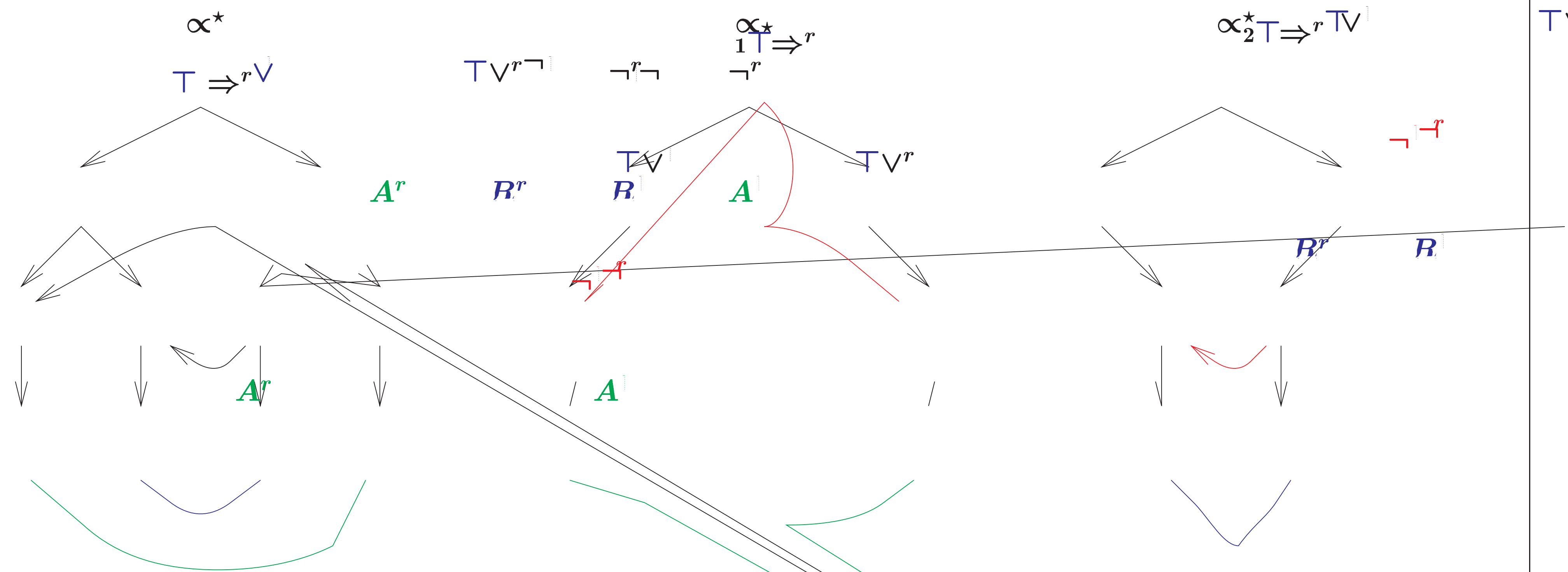
Basic Idea: Traverse reduc ion ordering $\propto^\star$ o cons ruc a sequen proof in LJ_{mc}

$$\propto^\star = \text{formula ree} + \text{Connec ions} + \text{subs i u ion cons rain s} \sqsubset$$



$\neg A \vdash \neg B,$

Proof Reconstruction II: β-split



$$\dfrac{\dfrac{\overline{A \vdash A}\ axiom}{\neg A, A \vdash}\ \neg l}{\boxed{\neg A \vdash \neg B, \neg A}}\ \neg r \qquad \dfrac{\dfrac{\overline{B \vdash B}\ axiom}{\neg B, B \vdash}\ \neg l}{\boxed{\neg B \vdash \neg B, \neg A}}\ \neg r$$

Demonstration I: Actual state of JProver

Realized componentes:

Input :	Formula as MetaPRL term
Connection Prover:	Proof search for propositional logic (intuitionistic or classical)
Reconstruction component :	Constructs sequent proofs in LK (classical logic), LJ_{mc} (multiply conclusioned), or LJ (single conclusioned)

– Switching between LJ_{mc} and LJ depends only on the selection strategy during the traversal of $\propto^\star$; (recall that LJ cannot polynomially simulate LJ_{mc})

– Cut-free proof ;ionstruction relies on complete redundancy deletion in $\propto^\star$.

Refiner: Not yet stand-alone; embedded into MetaPRL to use its operations

Demonstration II: Examples

Compare JProver to variants of tactic-based Simple Prover:

$A \vee \neg A$	cprover (tries magic as well)
$A \Rightarrow A$	prover (straightforward steps, goes into disjunctions as well)
$\neg\neg(A \vee \neg A)$	lprover (same as prover with stronger chaining and less control)
$\neg\neg(A \wedge B \vee \neg A \vee \neg B)$	(two interactions, picking $\neg A$, $\neg B$)
$S \wedge (\neg(T \Rightarrow R$	

WITCHING ETWEEN LJ_{mc} AND LJ

ynamic comple ion of reduc ion ordering $\propto^\star$ during proof recons ruc ion necessary

LJ_{mc}:

$$\frac{\Gamma, A \vdash}{\Gamma \vdash \neg A, \Delta}\ \neg r \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B, \Delta}\ \Rightarrow r$$

Selec ion s ra egy: $\neg r$ or $\Rightarrow r$ rule applicable in $\propto^\star$ only if Δ is redundan

LJ:

$$\frac{\Gamma, \neg A \vdash A}{\Gamma, \neg A \vdash C}\ \neg l \qquad \frac{\Gamma, A \Rightarrow B\ A \qquad \Gamma, B\ C}{\Gamma, A \Rightarrow B \vdash C}\ \Rightarrow l \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ \vee r1$$

Selec ion s ra egy: $\neg l$ rule applicable in $\propto^\star$ only if C is redundan

$\Rightarrow l$ rule applicable in $\propto^\star$ only if C is redundan for he proof of A $r1$ ru

$\rightsquigarrow$

requires comple e redundancy dele ion in $\propto^\star$

DISCUSSION

To do:

- Extend prover / reconstruction components to quantifiers $\rightsquigarrow$ easy
- Realize JProver as stand-alone